Uncertainties

Theory and practice for air kerma and HVL

Aino Tietäväinen

2.12.2024

Theory

Error is the difference between a measurement result (measurand) and its true value.

- Has a numeric value, which may include a sign
- Can be random or systematic
- Not precisely known
- Known systematic errors can be corrected.

Uncertainty defines the range within which the measurement error is estimated to lie.

- Describes the spread that can be associated with the measurement result

For example, the error in a meter's reading can be corrected using a calibration coefficient. However, the calibration coefficient also has an uncertainty that cannot be corrected.

Determining the uncertainty budget

1. **Constructing the model equation**
2. **Standard uncertainties**
3. **Combined standard uncertainty**
4. **Expanded measurement uncertainty** ("k=2")
5. **Presenting the uncertainty**

Model equation

- **Equation defining the components needed to calculate uncertainty**
  - Broader than the equation used to calculate the specific quantity.

- **Air Kerma Equation**: $K_a = N_{K,Q} \cdot k_{Tp} \cdot M$

- **Air Kerma Model Equation**: $K_a = N_{K,Q} \cdot k_{Tp} \cdot \underline{k_{dist} \cdot k_{lin} \cdot k_{en}} \cdot \ldots \cdot M$

Carefully consider the type of measurement for which you want to estimate uncertainty!

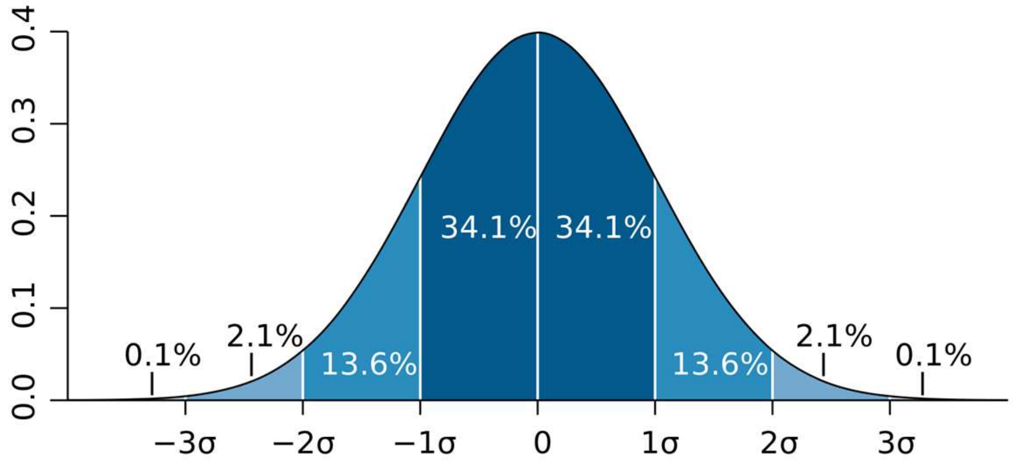
$\mathbf{K_a}$ = air kerma (mGy)
$\mathbf{N_{K,Q}}$ = calibration coefficient
$\mathbf{k_{Tp}}$ = temperature and pressure correction factor
$\mathbf{k_{dist}}$ = chamber positioning correction factor
$\mathbf{k_{lin}}$ = chamber nonlinearity correction factor
$\mathbf{k_{en}}$ = energy response correction factor
M = meter reading (mGy)

Determining the uncertainty budget

1. **Constructing the model equation**
2. **Standard uncertainties**
3. **Combined standard uncertainty**
4. **Expanded measurement uncertainty** ("k=2")
5. **Presenting the uncertainty**

Standard uncertainty, type A (u_A)

- Statistical estimation, repeated measurements
- The number of repetitions is at least *N*=10
- Probability distribution assumed normal.



https://fi.wikipedia.org/wiki/Hajontaluku

Standard deviation: $$s(x_i) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

$u_A = s(x_i)$: uncertainty for a single measurement.
Often expressed in %: $u_A = 100\% \cdot s(x_i)/|\bar{x}|$

Standard deviation of the mean:

$$s(\bar{x}) = \frac{1}{\sqrt{N}} s(x_i)$$

$u_A = s(\bar{x})$: uncertainty of the mean of *N* measurements. Often $u_A = 100\% \cdot s(\bar{x})/|\bar{x}|$

Standard uncertainty, type B (u_B)

- Estimated otherwise than statistically (e.g. manufacturer's specification etc.)

- Type A uncertainty is characterized by the standard deviation of the measurements (68% confidence level with a normal distribution).

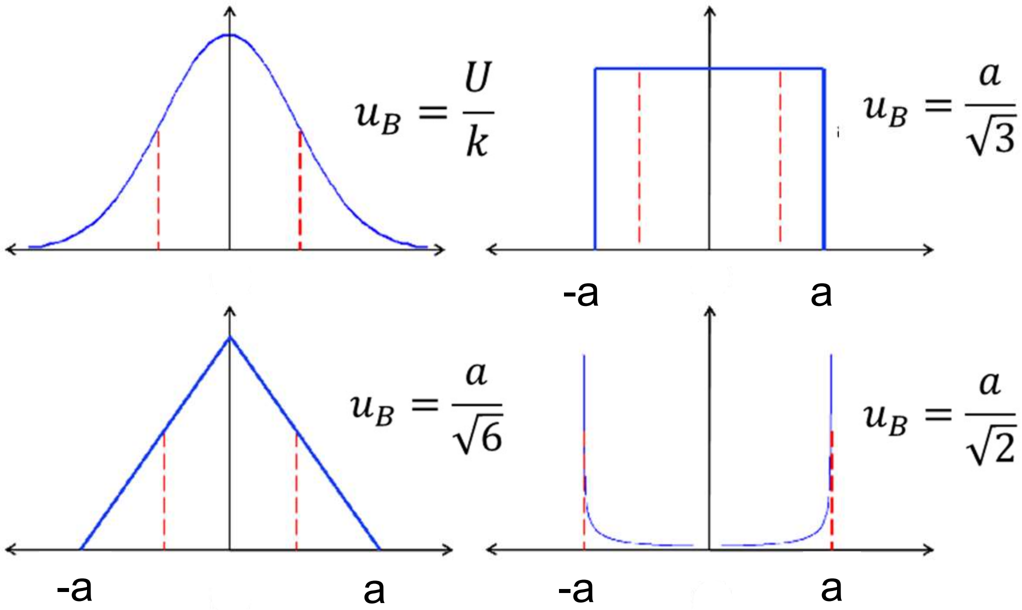
→ Type B uncertainties must be estimated so that they also correspond to standard deviations, i.e. they must be "on the same starting line" as type A uncertainties.

→ Assessment of range within which the uncertainty can vary (-a…+a) and probability distribution.

Standard uncertainty, type B (u_B)

Normal distribution
- Probability higher near the mean than at the extremes
- U = expanded uncertainty
- Denominator: coverage factor, k

Triangular distribution
- Probability is higher near the mean than at the extremes -a and +a
- Denominator = √6



Uniform/rectangular distribution
- Probability between -a and +a is constant
- Conservative (denominator = √3)

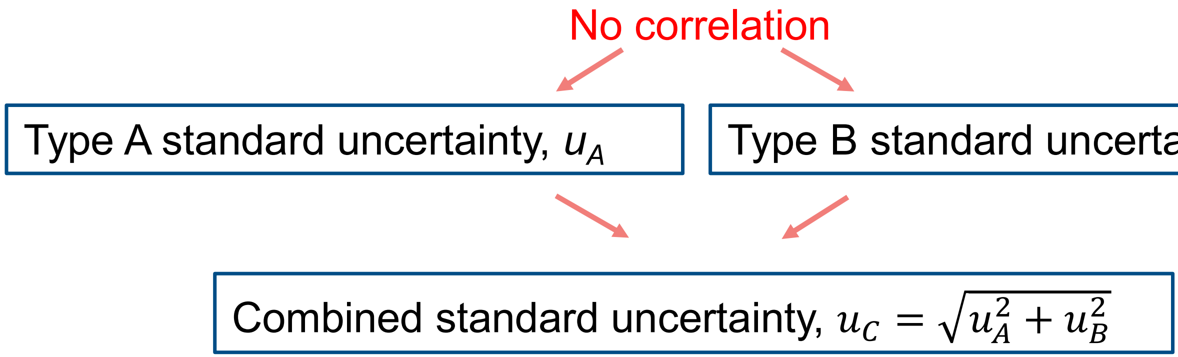
→ 1) Estimate maximum limits ±a, 2) estimate distribution, 3) calculate u_B.

Determining the uncertainty budget

1. **Constructing the model equation**
2. **Standard uncertainties**
3. **Combined standard uncertainty**
4. **Expanded measurement uncertainty** ("k=2")
5. **Presenting the uncertainty**

Combined standard uncertainty, u_C

- *Independent* type A and type B standard uncertainties u_A and u_B are combined quadratically according to the law of propagation of uncertainties.
- This gives the combined standard uncertainty, u_C.



Combined standard uncertainty, u_c

- Model equation:

$$y = f(x_1, x_2, x_3 \dots x_n)$$

<u>Law of propagation of uncertainty:</u>

- Input quantities x_i are independent:

$$u_C^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)$$

Standard *variance* (unc.^2)

Sensitivity coefficient, *c*

Combined standard *variance*

- Input quantities x_i are correlated:

$$u_C^2(y) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

Covariance

$-1 \leq r \leq 1$

$$u(x_i, x_j) = r(x_i, x_j)\, u(x_i)\, u(x_j)$$

Correlation coefficient, *r*

Combined standard uncertainty, u_c

No correlations assumed here.

1. Model equation: addition and subtraction

$$y = c_1 \cdot x_1 + c_2 \cdot x_2 + c_3 \cdot x_3 + \cdots$$

$$u_C(y) = \sqrt{c_1^2 \cdot u^2(x_1) + c_2^2 \cdot u^2(x_2) + c_3^2 \cdot u^2(x_3) + \cdots}$$

Absolute values!

2. Model equation: multiplication and division

$$y = x_1^{\alpha} \cdot x_2^{\beta} \cdot x_3^{\gamma} \cdot \cdots$$

$$\frac{u_C(y)}{|y|} = \sqrt{\alpha^2 \cdot \left(\frac{u(x_1)}{|x_1|}\right)^2 + \beta^2 \cdot \left(\frac{u(x_2)}{|x_2|}\right)^2 + \gamma^2 \cdot \left(\frac{u(x_3)}{|x_3|}\right)^2 + \cdots}$$

Relative values!

Sensitivity coefficient, *c*

T_0: reference temperature of 20°C in kelvin, 293,15 K
T: measured temperature (K)
t: measured temperature (°C)
p_0: reference pressure, 101,325 kPa
p: measured pressure (kPa)

$\frac{273{,}15+t}{T_0} \cdot \frac{p_0}{p}$ or $\frac{T}{T_0} \cdot \frac{p_0}{p}$ → use Kelvins!

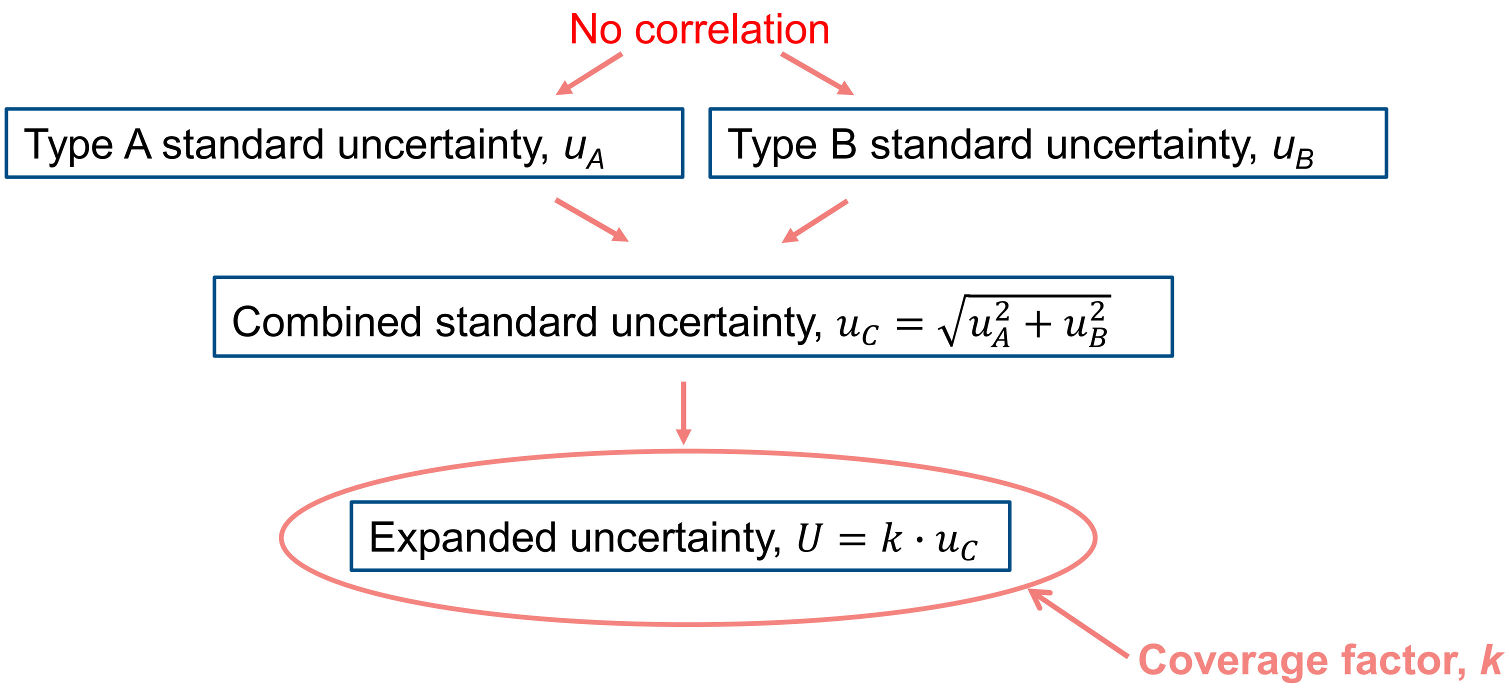
For air kerma: $K_a = N_{K,Q} \cdot k_{Tp} \cdot k_{dist} \cdot k_{lin} \cdot k_{en} \cdot \ldots \cdot M$

Determining the uncertainty budget

1. **Constructing the model equation**
2. **Standard uncertainties**
3. **Combined standard uncertainty**
4. **Expanded measurement uncertainty** ("k=2")
5. (**Presenting the uncertainty**)

Expanded uncertainty, *U*

- Combined standard uncertainty, u_C, is multiplied by coverage factor, *k* (integer, often *k*=2).



Expanded uncertainty, *U*

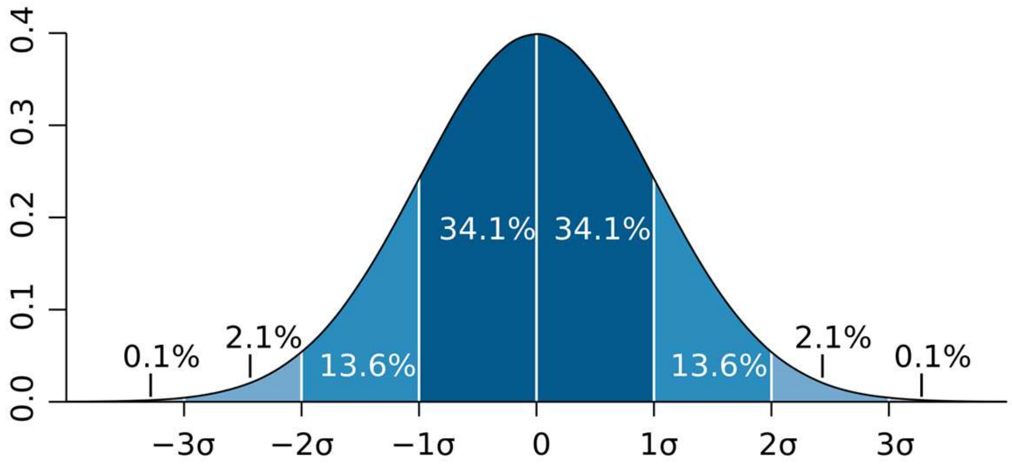
- The larger the coverage factor *k*, the larger the expanded uncertainty *U,* and the higher the confidence level

$$U = k \cdot u_C$$

k = 1: ~68% confidence level (u_C, for a normal distribution, approximately 68% of the measurement results will fall within this range of uncertainty).

k = 2: ~95% confidence level (the “typical case”)

k = 3: ~99% confidence level.

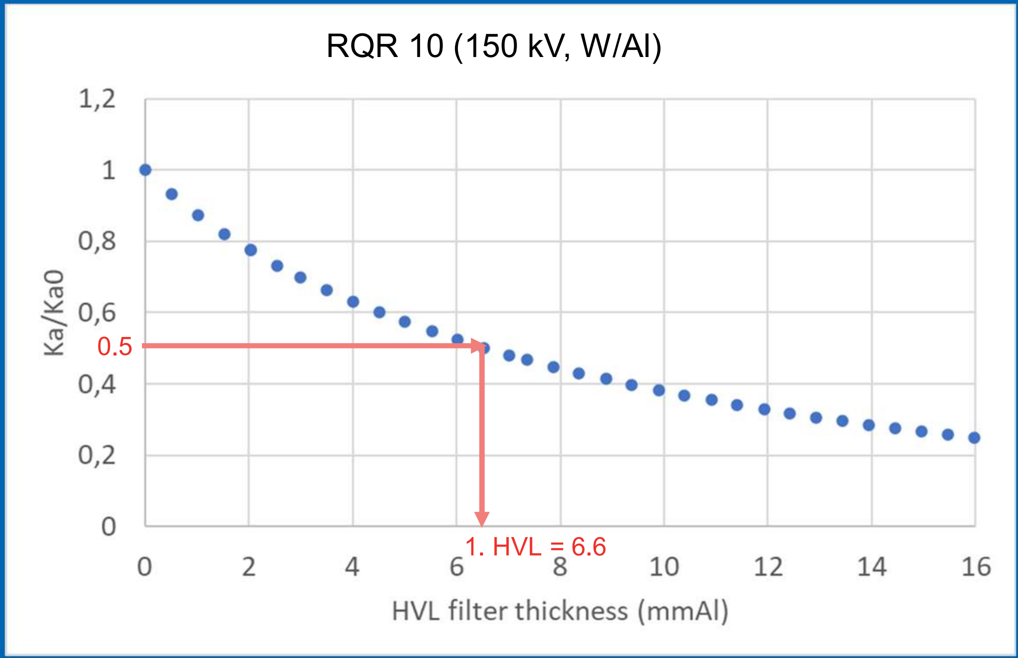


https://fi.wikipedia.org/wiki/Hajontaluku

Example: “The expanded uncertainty of the calibration coefficient with coverage factor *k* = *2* is estimated as 2.8% that corresponds to circa 95 % confidence level for normal distribution.”

Work in progress: Uncertainty of the first HVL

- Measurements conducted in narrow beam configuration (no scatter)
- Aluminum foils added to the X-ray beam
- Air kerma rate ($\dot{K}_a$) measured with an ionization chamber
- Thickness at which the air kerma rate halves $\dot{K}_a = \dot{K}_{a,0}/2$ is estimated ($\dot{K}_{a,0}$ is the air kerma rate without HVL filters).
  → First HVL (mmAl)



$$\frac{\dot{K}_a}{\dot{K}_{a,0}} = e^{-\mu} \quad \rightarrow \quad ln\left(\frac{\dot{K}_a}{\dot{K}_{a,0}}\right) = -\mu x$$

Linearization

stuk SÄTEILYTURVAKESKUS STRÅLSÄKERHETSCENTRALEN RADIATION AND NUCLEAR SAFETY AUTHORITY

TraMeXi

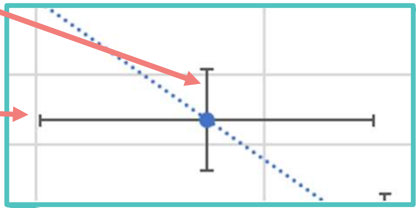
Uncertainty of the first HVL

More info: Katy Klauenberg *et al*. "The GUM perspective on straight-line errors-in-variables regression", Measurement, 2022 (187)

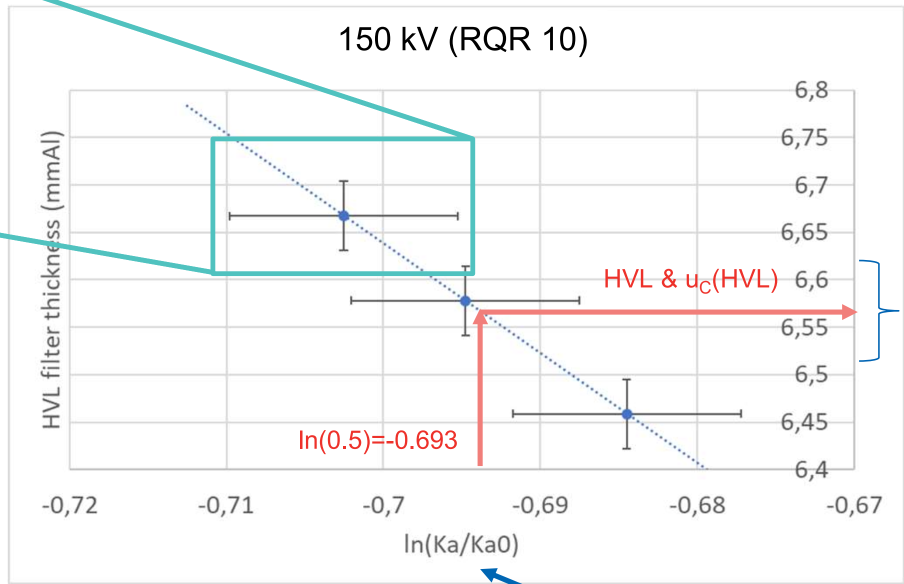
- HVL value <u>and it's uncertainty</u> can be estimated through **weighted total least squares regression (WTLS)**

u_C in y-axis (HVL filter thickness)

u_C in x-axis ($\ln(K_a/K_{a0})$)



x- and y-axis interchanged!



linearized

- ISO TS 28037, 2010, Determination and use of straight-line calibration functions
  - **Model equation** optimization problem (weighed minimization of residuals)!
  - **Correlations**: "Model for uncertainties and covariances associated with the x_i and the y_i" (Clause 10)
  - u_C related to filter thicknesses are correlated with each other, the u_C related to K_a measurements are correlated to each other, but no common sources of uncertainties between filter thickness and K_a measurement → no correlation.

Input: HVL measurements + matrix of uncertainties & covariances
Output: Slope, intercept and their uncertainties
Forward evaluation → HVL & its uncertainty

Uncertainty of the first HVL

Energy dependance

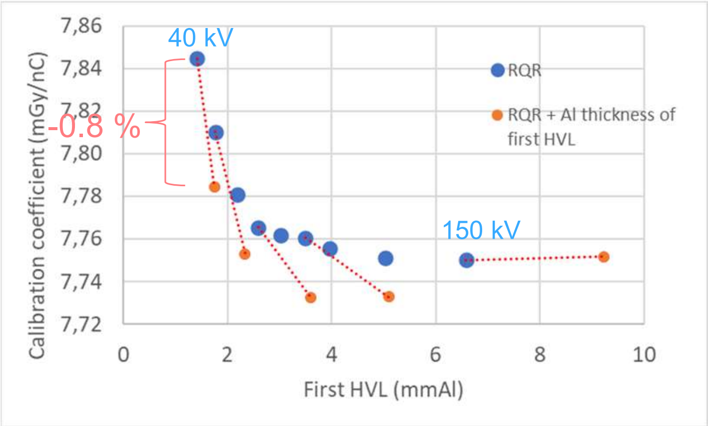
LOW energy qualities (I think)

- The spectrum of the X-ray beam changes when HVL filters are introduced to the beam.

→ Calibration coefficients change

→ Use IC with low energy dependance AND/OR correct calibration coefficient AND/OR estimate the uncertainty of the effect

Reference IC: Exradin A3



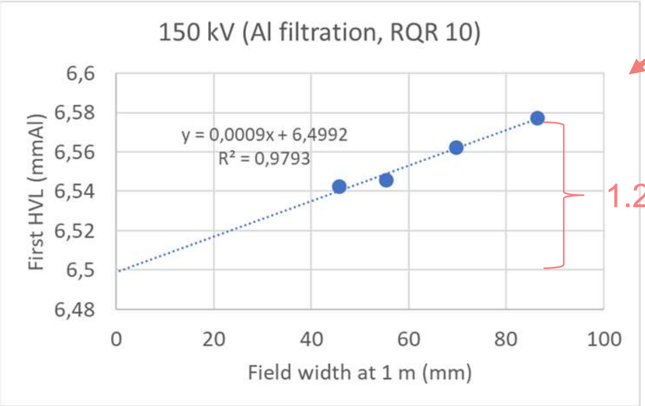
Zero-field correction

HIGH energy qualities, large field size & ICs.

- HVL is measured in narrow-beam configuration and defined in zero field size.

→ HVL changes with field size

→ Use as narrow field-size as possible AND/OR correct for the field size AND/OR estimate the uncertainty of the effect



Another WTLS regression…

1.2 % difference

Literature

- JCGM 100 (2008; GUM 1995 with minor corrections) Evaluation of measurement data – Guide to the expression of uncertainty in measurement (GUM)
- IAEA TRS-457 (2008) Dosimetry in diagnostic radiology: An international code of practice
- IAEA tecdoc 1585 (2008) Measurement uncertainty. A practical guide for secondary standards dosimetry laboratories

THANK YOU FOR YOUR ATTENTION!