Practical session: uncertainty calculation

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WHY we need dosimetry?

- Technical performance
  - Acceptance: system performs as specified
  - QC: continuous performance
- Validation of different dose displays and reports
  - KAP meter, K at Interventional Reference Point (IRP)
  - Dose in dicom header, Structured report (RDSR), dose mapping
- Patient dose, generic
  - Justification: benefit vs risk (averaged risk estimate)
  - Optimization: ALARA
    - DRLs = a tool
- Patient specific dose (e.g. pregnant woman)
  - Individual risk?

The purpose will also impact on the target uncertainty!

- 100%
- 50%
- 25%
- 15%
- 10%
- 7%
- 5%
- 3%
- 1%

(95%, k=2)

Practical example

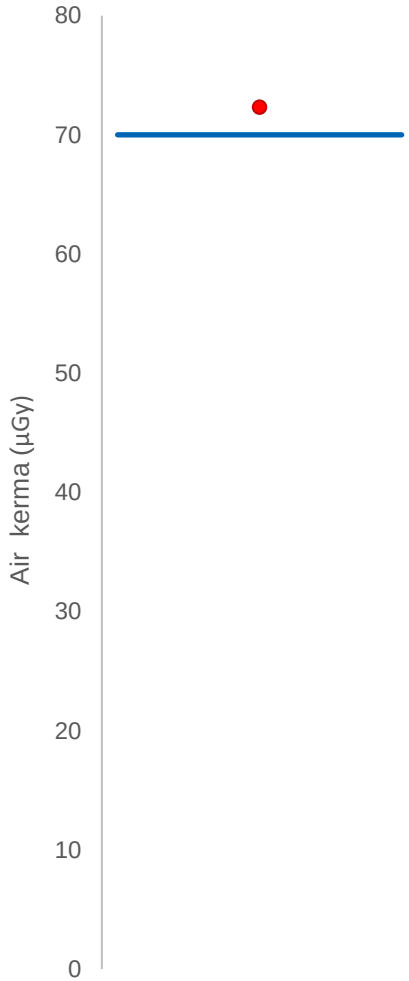
µGy

72.3

Last time it was 70 µGy.

The reference level is 70 µGy.

Is this result acceptable?



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Measurement result = measurand + uncertainty

- The uncertainty describes the **assumed variation** (distribution) of the values obtained for the measurand.
- **We cannot make any conclusions** based on our measurement results if we don’t know the uncertainty.

Uncertainty estimation steps

Step 1: Model function

Step 2: Uncertainty components

Step 3: Uncertainty budget

Step 4: Presentation of uncertainties

STEP 1: MODEL FUNCTION

Model function $y = f(x_1, x_2, \ldots, x_N)$

- For the whole measurement process
  - Not only the equation used for the calculations.
  - Includes also correction factors ≈ 1.
- *M* is the measurement result
- *N* is the calibration coefficient
  - That is given for the calibration conditions!
- *k*-factors cover different corrections needed because: measurement conditions ≠ calibration conditions.

$$K = M_Q \, N_{K,Q_0} \prod_i k_i$$

$$K = M_Q \, N_{K,Q0} \, k_Q k_T k_p k_{field} k_{other}$$

IEC 61674

Table A.1 – Estimation of COMBINED STANDARD UNCERTAINTY for dosimeter performance

PERFORMANCE CHARACTERISTIC	Subclause	Relative STANDARD UNCERTAINTY[a] %
Calibration factor[b]		±2,89
Linearity	5.1	±1,15
Repeatability	5.2	±0,58
RESOLUTION of reading	5.3	±0,58
STABILIZATION TIME	5.4	±1,15
Long term stability	5.6.1	±1,15
Accumulated dose stability	5.6.2	±0,58
RADIATION QUALITY	6.2	±2,89
AIR KERMA RATE	6.3	±1,15
Incidence of radiation	6.4	±1,73
Operating voltage	6.5	±1,15
Air pressure	6.6	±1,15
Temperature and humidity	6.8	±1,73
Electromagnetic compatibility	6.9	±2,89
Field size	6.10	±1,73
COMBINED STANDARD UNCERTAINTY		±6,6

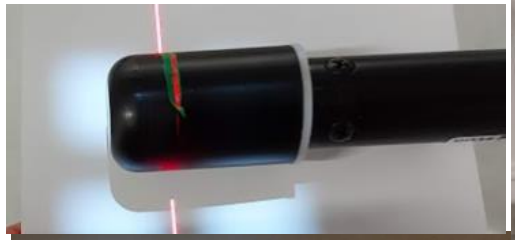
[a] Relative STANDARD UNCERTAINTY assuming that there is no additional information about the PROBABILITY DISTRIBUTION of the PERFORMANCE CHARACTERISTIC within the allowed interval other than it has an uniform distribution, i.e. 0,577 *L* for symmetric limits.

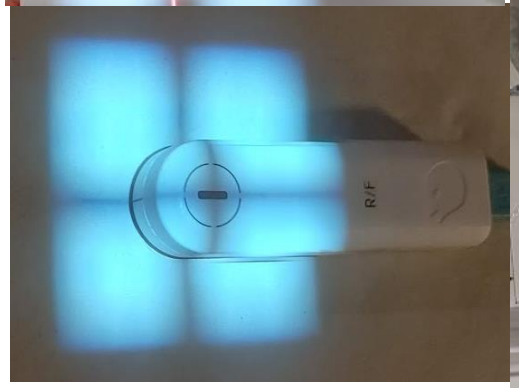
[b] Although no requirement on the accuracy of the calibration factor is laid down in this document, a maximum error of the calibration factor is included here and assumed to be ±5 %. A uniform distribution is also assumed.

Dosimeter performance related uncertainties

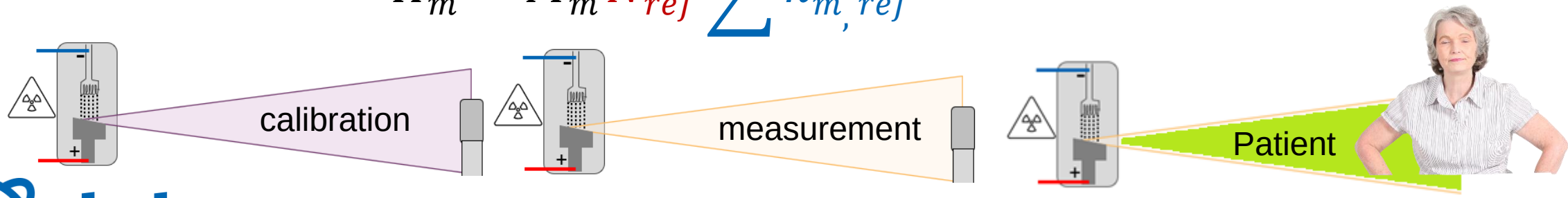
Measurement conditions

- Measurement conditions will impact the uncertainty.
- Geometry
  - Positioning of dosimeter
    - Effective plane/point
  - Field size
  - Distance
- Scatter
- Radiation quality
- Environmental conditions
- Repetitions





$$K_m = M_m N_{ref} \sum k_{m,ref}$$

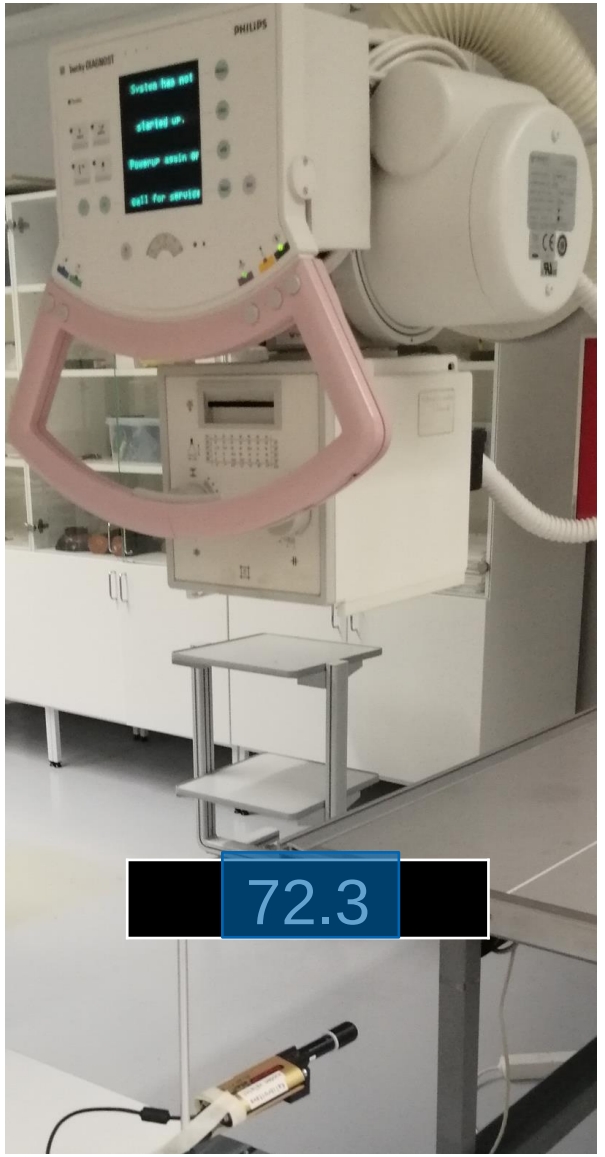


Practical example

- You are measuring air kerma
  - displayed air kerma value(s): in average 72.3 µGy

  - Radiation quality: 75 kV, 5 mmAl (HVL = 3.7 mmAl)
  - Measurement distance: 1 m = FCD
  - Field size @1 m: A = 15 cm*15 cm
  - Temperature t = 23℃
  - Air pressure p = 100.1 kPa

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{other}$$



Calibration certificate

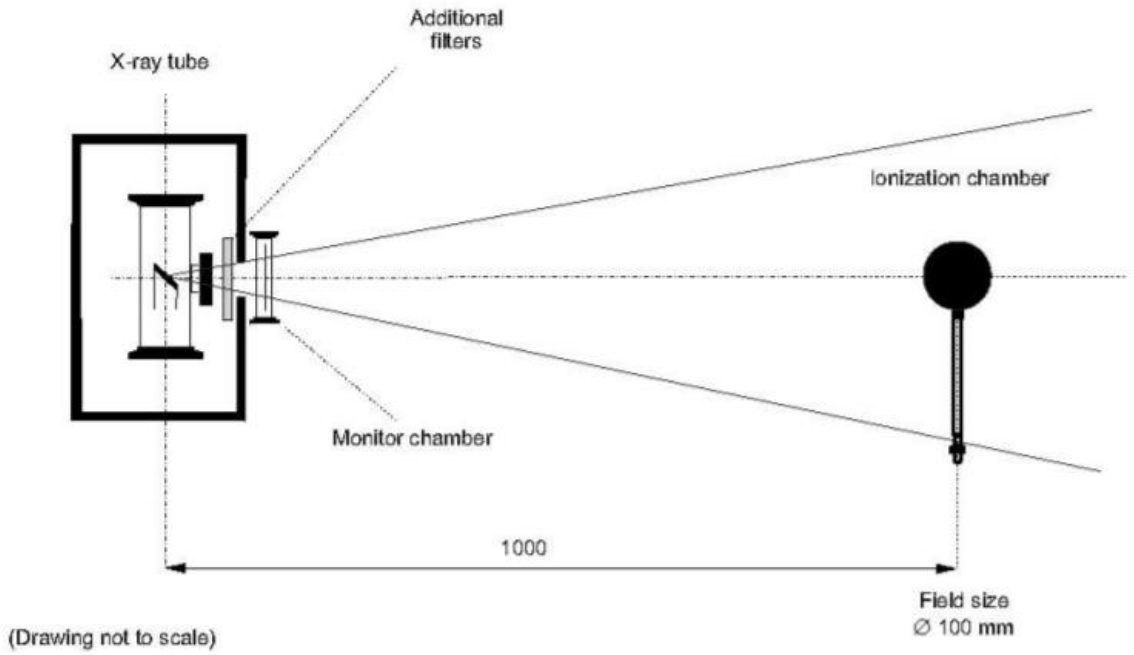


FIG. 1. Set-up for calibration in terms of air kerma for diagnostic radiology X-ray beams

Figure from ssdl.iaea.org: IAEA DOLP.013: Appendix 2

Radiation quality	Tube voltage (kV)	Total filtration (mmAl)	Half-value layer (mmAl)	Calibration coefficient
RQR 2	40	2.44	1.39	0.976
RQR 3	50	2.44	1.73	0.973
RQR 4	60	2.69	2.11	0.971
RQR 5	70	2.88	2.54	0.972
RQR 6	80	3.15	2.88	0.975
RQR 7	90	3.39	3.43	0.978
RQR 8	100	3.52	3.91	0.98
RQR 9	120	3.90	4.95	0.981
RQR 10	150	4.72	6.55	0.982

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{other}$$

The uncertainty of calibration coefficient: 2.8% .
The uncertainty is relative combined standard uncertainty, with a coverage factor of k = 2, which for a normal distribution corresponds to a level of confidence of approximately 95%

Calibration

- air kerma value(s):
  - @STUK: 0.5 mGy/s*30s = 15 mGy

- Radiation quality: IEC RQR-set
- Measurement distance: 1 m = FCD
- Field size @1 m: Ø = 10 cm
- Temperature t = 19℃
- Air pressure p = 102.1 kPa

=> Calibration coefficient given in NTP.

- Internal bias voltage not known

Measurement

- air kerma value(s): 72.3 µGy
  - E.g. 0.72 mGy/s* 0.1 s

- Radiation quality: 75 kV, 5 mmAl (HVL = 3.7 mmAl)
- Measurement distance: 1 m = FCD
- Field size @1 m: A = 15 cm*15 cm
- Temperature t = 23℃
- Air pressure p = 100.1 kPa
- Internal bias voltage not known

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{other}$$

Incident air kerma calculations

Example case:
1 mGy measured with 10 mAs @1m, RQ2+110 kV

- Measured air kerma, K_a, mGy at specific tube current time product, Q, mAs

$\Rightarrow$ Tube output, Y, mGy/mAs

$$Y(FDD) = \frac{K_a(FDD)}{Q}$$

$$= \frac{1.00\ mGy}{10\ mAs} = 0.1\ mGy/mAs$$

- Tube output Y_d measured or interpolated for the correct radiation quality (@FDD)
- Air kerma, K_a with tube current time product Q_p used for a patient p

$$K_{a,Qp}(FDD) = Y_m(FDD) \cdot Q_p$$

$$= 0.11 \frac{mGy}{mAs} \cdot 23 mAs$$

=2.53 mGy

- Distance correction for the patient surface, S (or any other relevant distance)

$$K_{a,Qp,FSD} = K_{a,Qp,FDD} \cdot \left(\frac{FDD}{FSD}\right)^2$$

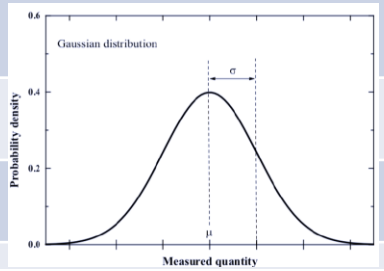
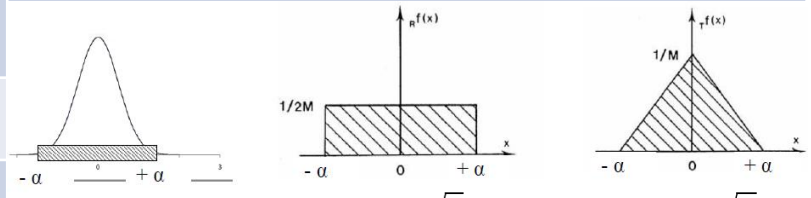
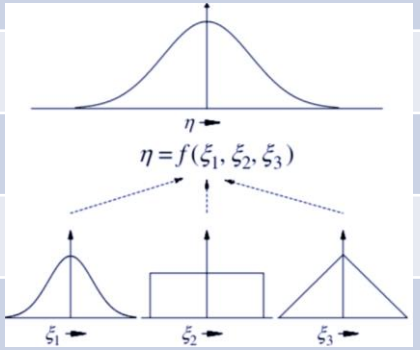
$$= 2.53\ mGy \cdot \left(\frac{1\ m}{2\ m}\right)^2$$

= 0.6325 mGy

A- and B-type

STEP 2: UNCERTAINTY COMPONENTS

Uncertainty components

Component	Parameter	A-type (1 std, %)	B-type (1 std, %)
Displayed value deviation	M_Q		
Displayed value resolution and readout	M_Q		
Calibration coefficient	$N_{K,Q0}$		
Long term stability	$N_{K,Q0}$	$s(x_i) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$	$p = 3$ (~99.7% c.l.) $\quad u_B = \frac{a}{3}$ (~ SD)
Radiation quality	k_Q	$s(\bar{x}) = \frac{1}{\sqrt{N}} s(x_i)$	$p = \sqrt{3} \quad u_B = \frac{a}{\sqrt{3}}$
Temperature	k_T	$u_A = s(\bar{x})$	$p = \sqrt{6} \quad u_B = \frac{a}{\sqrt{6}}$
Air pressure	k_p		
Radiation field and positioning	k_{field}		
Other: leakage, operating voltage…	k_{others}		

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{others}$$

Uncertainty budget: IEC values

Component	(1 std, %)
Repeatability (A-type)	0.58
Resolution	0.58
Displayed value linearity, air kerma rate	1.15, 1.15
Stabilization time	1.15
Calibration coefficient	2.89
Long term stability, accumulated dose stability	1.15, 0.58
Radiation quality	2.89
Temperature	1.73
Air pressure	1.15
Incidence of radiation	1.73
Field size	1.73
Electromagnetic compatibility	2.89
Operating voltage	1.15
Standard uncertainty (k=1, %)	**6.6**
Expanded uncertainty (k=2, %)	**13.1**

Component	(%)
Repeatability	0.58
Resolution	0.58
Linearity	1.63
Stabilization	1.15
Calibration coef.	2.89
Long term stability	1.29
Radiation quality	2.89
Temperature	1.73
Air pressure	1.15
Incidence of rad.	1.73
Field size	1.73
EC	2.89
Operating voltage	1.15
u (k=1, %)	**6.6**
U (k=2, %)	**13.1**

Only the 3 oranges
=> 5% (k=1)

Repeatability

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{other}$$

$$s(\bar{x}) = \frac{1}{\sqrt{N}} s(x_i) \qquad u_A = s(\bar{x})$$

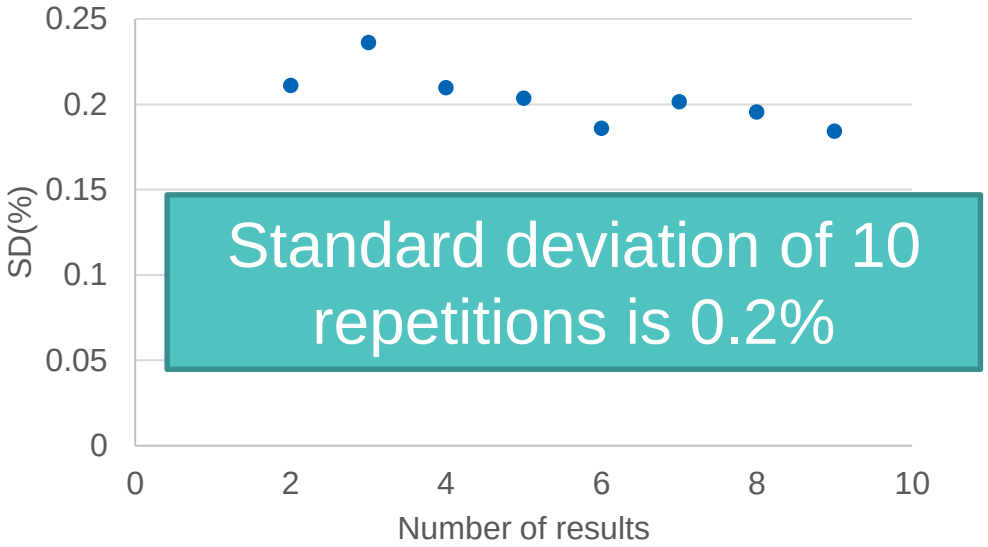
72.3
u = 0.2%

72.3
72.2
72.5
u = 0.12%

u?

u =0.2%/√3=0.12%

72.3
72.2
72.5
72.1
72.2
72.4
72.3
72.5
72.2
72.3
u = 0.06%



- Make enough repetitions at least once to make a good estimation of the distribution.
- Then smaller number of repetitions can be justified.

Example of today

- Repetitions:
  - 70 kV, 0 mmCu
  - 10 cm*10 cm
- Measured and displayed air kerma values (mGy) or KAP values.

	IC (mGy)	XMM (mGy)	KAP (mGy*cm²)
	0.4263	0.3917	57
	0.4269	0.3916	56
	0.4271	0.3913	57
	0.4273	0.3920	56
	0.4261	0.3916	58
	0.4265	0.3917	56
	0.4269	0.3918	57
	0.4271	0.3912	56
	0.4274	0.3922	57
	0.4270	0.3916	56
Mean (mGy)	**0.4269**	**0.3917**	**56.6**
SD (%)	**0.1**	**0.08**	**1.2**
u(%) n=3	**0.06**	**0.04**	
u(%) n=10	**0.03**	**0.02**	

u?

Resolution

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{other}$$

72.3
72.2
72.5
72.1
72.2
72.4
72.3
72.5
72.2
72.3

72.3

Is this actually

u?

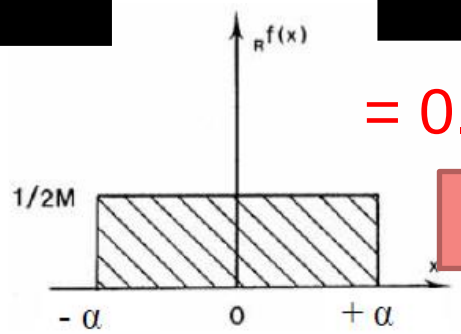
KAP (mGy*cm²)
57

72.251 OR 72.349

= 0.5/57*100%

= 0.9%=> u=0.5%

u?

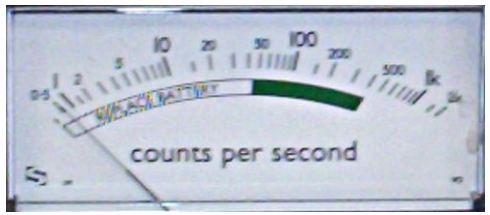


Rectangular

= 0.05/72.3*100% = 0.07%

u =0.07%/√3=0.04%

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{other}$$



Calibration coefficient

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{others}$$

Calibration or adjustment does not remove the uncertainty!

- Uncertainty for the calibration coefficient from the calibration certificate.
- Here it is 2.8% (k=2)
- normal distribution, B-type

u = 2.8%/2=1.4%

u?

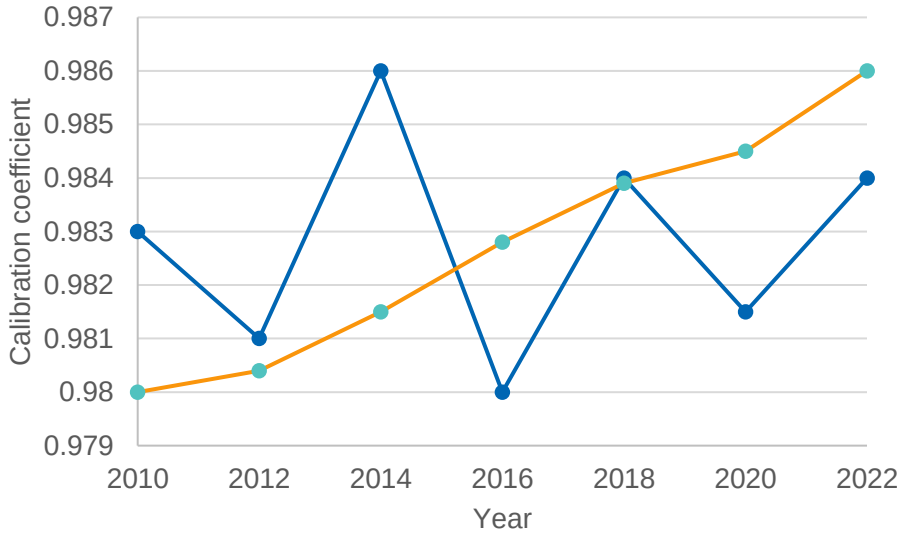
- Long term stability can be estimated based on repeated calibrations or based on quality control measurements.
  - Deviation (A) or drift (B)

SD = 0.2% or 0.6%√3=0.4%?

?

Calibration certificate:

The uncertainty of calibration coefficient: 2.8%. The uncertainty is relative combined standard uncertainty, with a coverage factor of k = 2, which for a normal distribution corresponds to a level of confidence of approximately 95%



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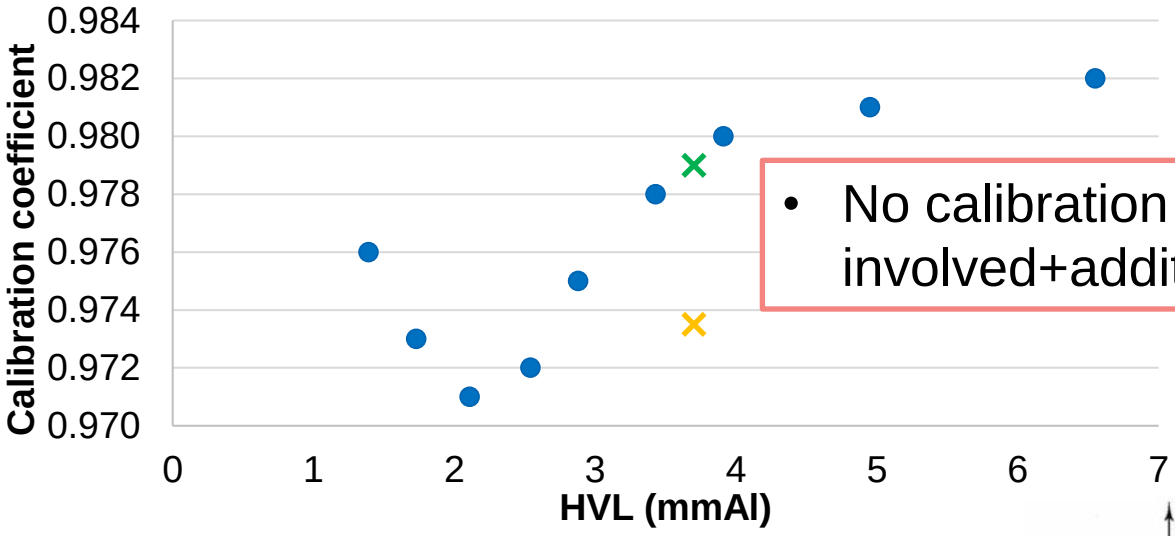
Calibration coefficient $K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{others}$

- Radiation quality (RQ) in the measurement:
  - ❑ 75 kV, 5 mmAl (HVL = 3.7 mmAl)
- How to select the correct calibration coefficient?
- What is the uncertainty?

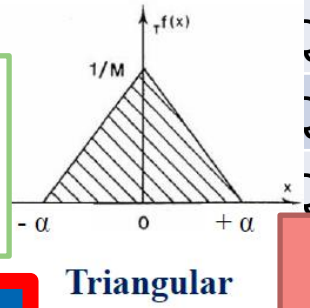


Calibration certificate:

Radiation quality	Tube voltage (kV)	Total filtration (mmAl)	Half-value layer (mmAl)	Calibration coefficient
RQR 2	40	2.44	1.39	0.976
			.73	0.973
			2.11	0.971
RQR 5	70	2.88	2.54	0.972
RQR 6	80	3.15	2.88	0.975
QR 7	90	3.39	3.43	0.978
QR 8	100	3.52	3.91	0.98
QR 9	120	3.90	4.95	0.981
			55	0.982

- No calibration coefficient: ~2% error is always involved+additional error related to RQ variation!

Impacting factors:
- Energy dependence of the response
- RQ: calibration vs. hospital
- RQ specifier and calibration coefficient selection



Triangular

?

The best-case scenario
u = 1%/√6≈0.4%

Environmental conditions

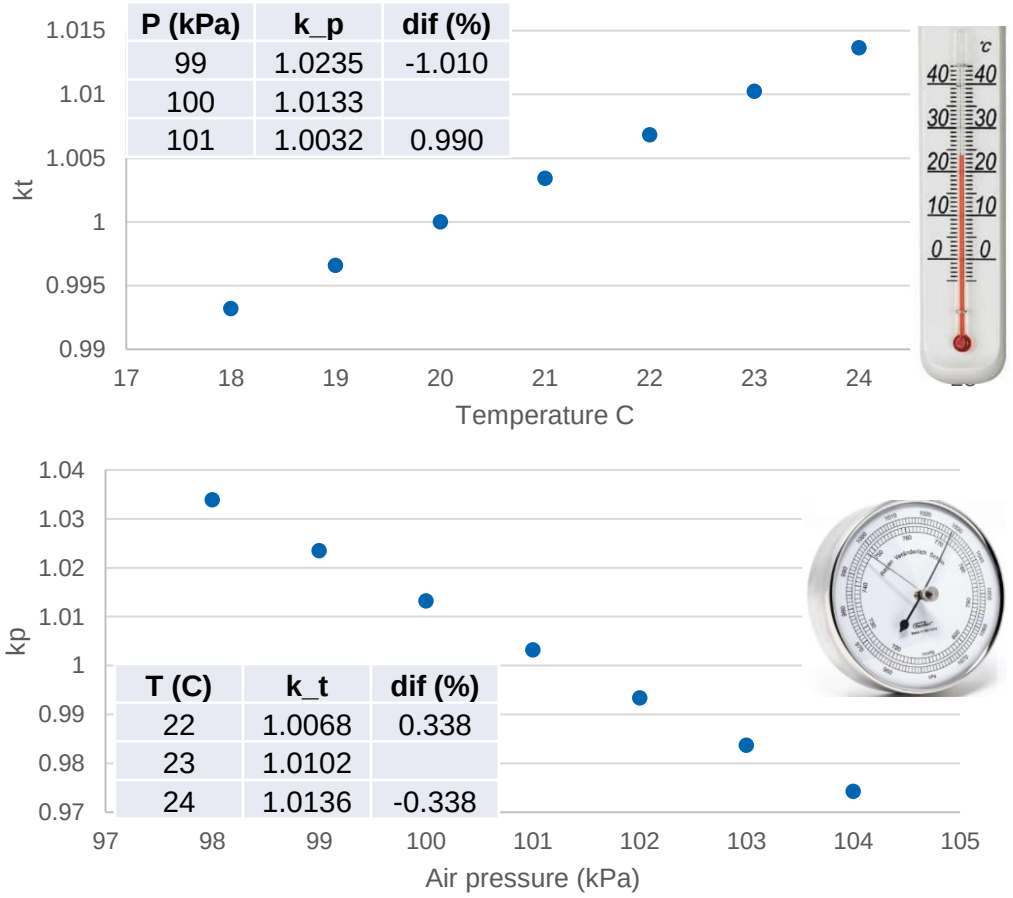
$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{others}$$

- Measurement of T and p
  - Calibration+all other components (e.g. repeatability, location)
    - Δ1°C=> a=0.3%, rec
    - Δ 1 kPa => a=1%, rec
  - Note also change of response (IEC)
  - No correction=> error, ~uncertainty
    - E.g. temperature: ±1% triangular=> 0.4%
    - E.g. air pressure: ± 3% rectangular=> 1.7%
  - Note: also humidity and electromagnetic fields

u?

u =0.3%/√3=0.17%

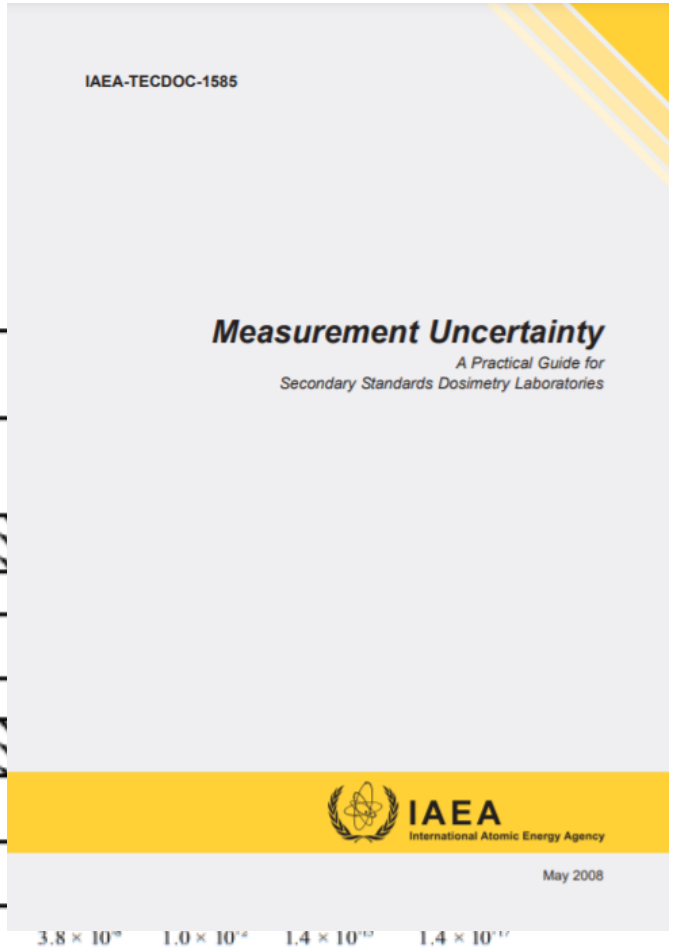
u =1%/√3=0.58%



Excellent example of uncertainty estimation for air density

TABLE 2. UNCERTAINTY BUDGET FOR EXAMPLE 5 MEASURING AIR DENSITY (FINAL VERSION)

Note	Quantity, Source of uncertainty	Value of quantity	Expanded uncertainty	Uncertainty type	Confidence level	Coverage factor	Standard uncertainty	Sensitivity coefficient	Uncertainty component	Effective degrees of freedom
			U_i			k	u_i	c_i	$\lvert c_i u_i \rvert$	ν_{eff}
Pressure measurement		102.30 kPa								
1	Barometer calibration		0.1 kPa	Type B	95%	2	0.05%	1.00	0.05%	30
2	Uncertainty of mean reading		0.03 kPa	Type A	68%	1	0.03%	1.00	0.03%	9
3	Barometer resolution		0.005 kPa	Type B	100%	1.73	0.003%	0.32	0.001%	100
Temperature measurement		23.4 °C								
4	Uncertainty of mean reading		-	Type A	-	-	-	-	-	-
5	Thermometer calibration		0.5 °C	Type B	95%	2	1.07%	0.08	0.08%	30
6	Thermometer resolution		0.1 °C	Type B	100%	1.73	0.25%	0.08	0.02%	100
Air density correction		1.0019								
7	Combined uncertainty		0.21%	Combined	95%	2.0	0.10%			59

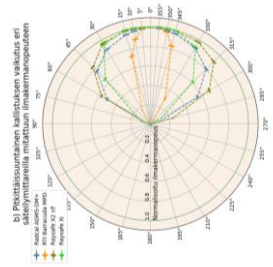


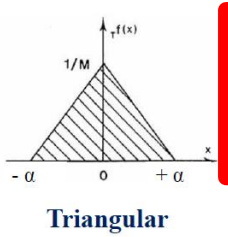
Positioning

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{others}$$

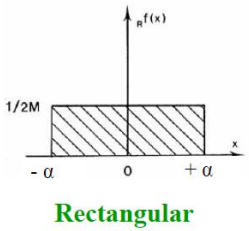
- Positioning as in calibration (note markings)
- No tilt Master thesis of Viivi Valkama
- Stem-effect
- Positioning at specific distance or for a patient
  - e.g. 1 m ± 1cm.
  - impact $\frac{1}{r^2}$
    - 1 cm @1m ~2%, triangular => 0.8%
- Field uniformity and me
  - E.g. 1% uniformity
  - Impact depends on dosimeter…

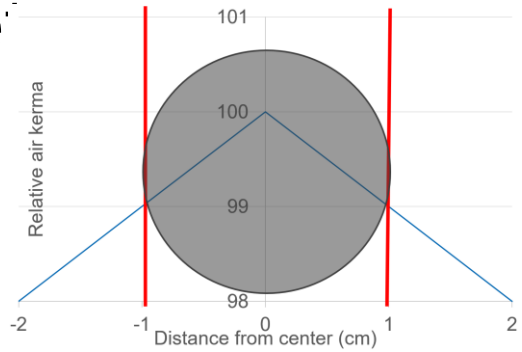
u = 2%/√6= 0.82%

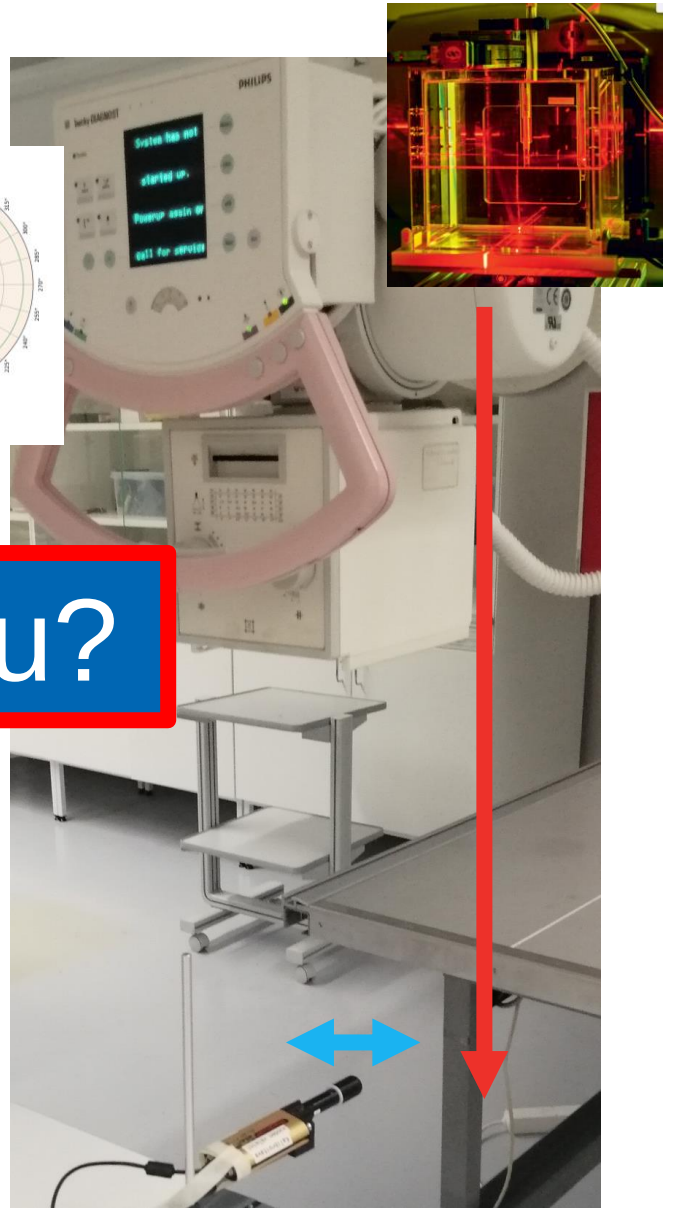




u?





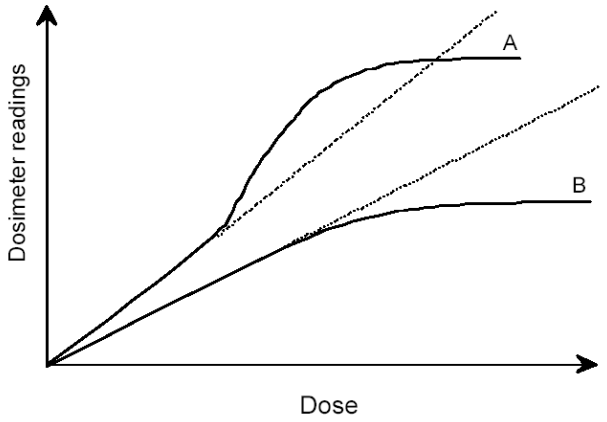


u =0.5%/√3=0.29%

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{others}$$

Other components:

- Leakage current, background: *M*
  - Impact depends on the measurand (absolute value)
- Stabilization time (enough): *M*
- Linearity (air kerma) k_{lin}
  - Air kerma(rate): calibration vrs. hospital
  - Data from calibration, tec. Specifications



- Electrometer calibration (included or separate) N_{el} or k_{el}
- The same operating voltage is applied=> small uncertainty $k_{rec} k_{pol}$

Incident air kerma calculations

- What about the uncertainties related to tube current time product display and X-ray system stability:
  - u = 2%/√3=1.15%
- Distance correction: dosimeter positioning vs. patient distance e.g. 1 m ± 1cm.
  - Calculated on earlier.
- Field size impact (not corrected):
  - u = 4%/√3=2.3%
- Scatter e.g., from the table
  - to be included or not?
- Attenuation e.g., due to table
  - Should be corrected, otherwise this might be the only uncertainty component which matters.

?

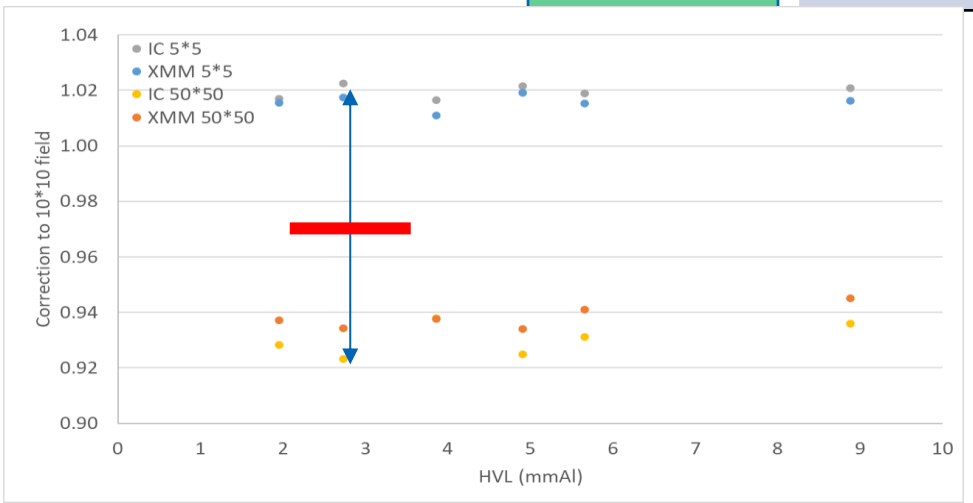
?

$$Y(FDD) = \frac{K_a(FDD)}{Q}$$

$$K_{a,Qp}(FDD) = Y_m(FDD) \cdot Q_p$$

$$K_{a,Qp,FSD} = K_{a,Qp,FDD} \cdot \left(\frac{FDD}{FSD}\right)^2$$

Component	u (%)
Repeatability	
Resolution	
Linearity	
Stabilization	
Long term stability	
Radiation quality	
Temperature	
Air pressure	
Field uniformity	



Combining uncertainties

STEP 3: UNCERTAINTY BUDGET

Uncertainty budget

Uncorrelated uncertainties

$$K = M_Q N_{K,Q0} k_Q k_T k_p k_{field} k_{others}$$

$u = \sqrt{(a^2+b^2+c^2....)}$

Component	IEC	Perfect	Typical
Repeatability	0.58	0.1	0.3
Resolution	0.58	0.04	0.5
Linearity	1.63	0.1	0.5
Stabilization	1.15	0	0.5
Calibration coef.	2.89	1.4	2.89
Long term stability	1.29	0.2	0.4
Radiation quality	2.89	0.4	2.89
Temperature	1.73	0.2	0.4
Air pressure	1.15	0.6	1.7
Incidence of rad.	1.73	0.1	0.3
Field size	1.73	0.3	0.5
EC	2.89	1	2.89
Operating voltage	1.15	0.1	1.15
u (k=1, %)	**6.6**	**1.9**	**5.5**
Field uniformity	0.29	0.29	0.29
positioning	0.82	0.82	0.82
X-ray system, mAs	1.15	1.15	1.15
Field size	2.3	2.3	2.3
u (k=1, %)	**7.1**	**3.3**	**6.2**
U (k=2, %)	**14.2**	**6.7**	**12.4**

U =2*u

Rounding, significant digits

STEP 4: PRESENTATION OF UNCERTAINTIES

Reporting uncertainties

1. Decide number of digits for the uncertainty (recom. 2), round up

Uncertainty ± 0.34456789 or 1.343118339% (k=2, 95%)

➢ Uncertainty ± 0.35 or 1.4% (k=2, 95%)

U = 5.547% => 5.6% or
U=4.01 µGy=>4.0 µGy

2. Select number of digits for the calibration coefficiet, normal rounding

Calibration coefficient: 25.654321 mGy/nC

➢ Calibration coefficient: 25.65 mGy/nC

K_a = 72.30 µGy => 72.3 µGy

3. Select format for your report!

- CC = (25.65 ± 0.35) mGy/nC (k=2, 95%)

K_a = (72.3 ± 4.0) µGy (k=2, 95%)

K_a = 72.3 µGy ± 5.5% (k=2, 95%)

…actually

CC = (25.65 ± 0.35) mGy/nC , where the number following the symbol ± is the numerical value of (an expanded uncertainty) U = ku_c, with U determined from (a combined standard uncertainty) u_c = 0,17 mGy/nC and (a coverage factor) k = 2 and confidence of 95 percent.

Practical example

µGy

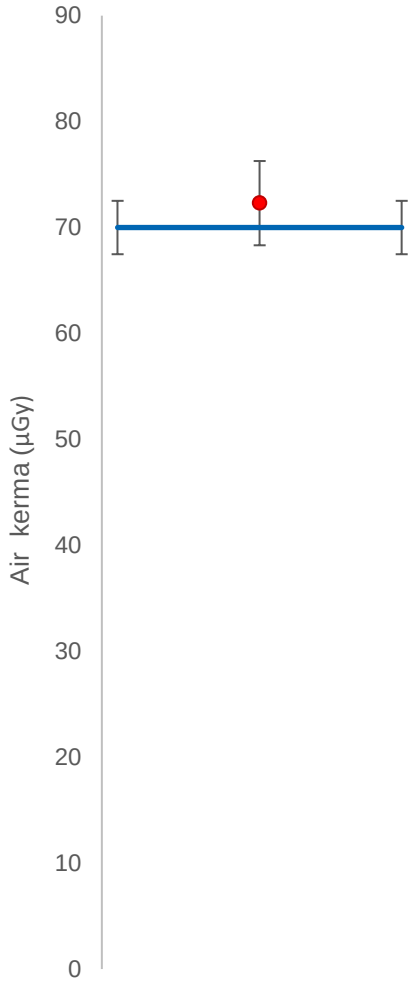
72.3

K_a = (72.3 ± 4.9) µGy (k=2, 95%)

Last time it was 70 µGy.

The reference level is 70 µGy.

Is this result acceptable?



stuk
SÄTEILYTURVAKESKUS
STRÅLSÄKERHETSCENTRALEN
RADIATION AND NUCLEAR SAFETY AUTHORITY

Take home message

- Measurement results = measurand + uncertainty
- The estimated values is linked with the method.
- The largest components dominate.

”The measurement chain is as weak as the weakest link (highest uncertainty) ”.

References

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- Bell, Stephanie 2003 Measurement Good Practice Guide no.11, National Physical Laboratory
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Thank you for your attention!

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